S.-J. Xu

Background and Motivation

Results on total edge dominations

Results on semitotal edge dominations

Complexity and characterizations of edge-related dominations on graphs

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joint work with Xianyue Li, Zhuo Pan, Yu Yang

Outlines

1. Background and Motivation
2. Results on total edge dominations
3. Results on semitotal edge dominations

Background for vertex version of domination

- A dominating set for a graph $G = (V, E)$ is a subset $D \subseteq V$ such that every vertex not in D is adjacent to at least one vertex in D.

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- compared with the concept of the vertex cover.
- The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for G.
- The domination problem was studied from the 1950s onwards, but the rate of research on domination significantly increased in the mid-1970s.
- The DOMINATING-SET problem concerns testing whether $\gamma(G) \leqslant k$ for a given graph G and input integer k.

- C. Berge, Theory of graphs and its applications, Methuen, London, 1958.
- O. Ore, Theory of graphs, Amer. Math. Soc. Colloq. Publ. 38, Providence, RI, (1962) 206-211.

Motivation for vertex version

- In 1972, Karp proved the VERTEX-COVER problem to be NP-complete. This had immediate implications for NP-completeness of the DOMINATING-SET problem.[Michael & David, 1979]
- Dominating sets are of practical interest in several areas. In wireless networking, dominating sets are used to find efficient routes within ad-hoc mobile networks. They have also been used in document summarization, and in designing secure systems for electrical grids.
- The wide variety of domination parameters that can be defined in accordance with various demands of real applications.

- Michael R. Garey, David S. Johnson (1979), Computers and Intractability: A Guide to the Theory of NP-Completeness, W. H. Freeman, ISBN 0-7167-1045-5, p. 190, problem GT2.
- Richard M. Karp (1972). "Reducibility Among Combinatorial Problems". In R. E. Miller and J. W. Thatcher (editors). Complexity of Computer Computations. New York: Plenum. pp. 85 C103.

Various types of problems on dominations

Generally, the following types of problems are considered in the field of domination in graphs [Vaidya & Pandit, 2014]:

(1) to introduce new types of dominating models;

(2) to determine bounds in terms of various graph parameters;

(3) to obtain the exact domination number for some graphs or graph families;

(4) to study the algorithmic and complexity results for particular dominating parameters;

(5) to characterize the graphs with certain dominating parameters;

(6) to study on domination-critical graphs;

(7) ······

• S. K. Vaidya and R. M. Pandit, Edge Domination in Some Path and Cycle Related Graphs, ISRN Discrete Mathematics, Volume 2014, Article ID 975812.

Edge version of domination

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- Compared with the concept of the edge cover.
- Edge domination number $\gamma'(G)$: the minimum cardinality among all edge dominating sets.
- The EDGE-DOMINATING-SET problem is to test whether $\gamma'(G) \leqslant k$ for an input graph G and input integer k.

S. Mitchell and S.T. Hedetniemi, *Edge domination in trees*, Congr. Numer. 19 (1977), 489-509.

Connections of edge version with well-known problems

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    - It is easily proved that the size, i.e., $\gamma'(G)$, of minimum edge dominating sets is equal to the size of minimum independent edge dominating sets or minimum maximal matchings, i.e., $\gamma'(G)=\min\{|M| \mid M$ is a maximal matching$\}$.

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    - Saturation number $s(G)$ in chemical graph theory =min$\{|M| \mid M$ is a maximal matching$\}=\gamma'(G)$. (chemical background: monomer-dimer = matching, pure dimer arrangement = perfect matching)

J.D. Horton, K. Kilakos, Minimum edge dominating sets, SIAM J.Discrete Math. 6 (3) (1993) 375-387.

NP-completeness for the EDGE-DOMINATING-SET problem

• The EDGE-DOMINATING-SET problem is NP-complete even when restricted to planar or bipartite graph of maximum degree 3. [Yannakakis and Gavril, 1980]

The EDGE-DOMINATING-SET problem is NP-complete for planar bipartite graphs, their subdivision, line graph, and total graph, perfect claw-free graphs, and planar cubic graph. [Horton, Kilakos, 1993]

[• There is a $O(V^2E)$ time algorithm to find a maximum matching or a maximum weight matching in a general graph that is not bipartite; it is due to Jack Edmonds, is called the paths, trees, and flowers method or simply Edmonds' algorithm.]

M. Yannakakis and F. Gavril, *Edge dominating sets in graphs*, SIAM Journal on Applied Mathematics 38(3) (1980) 364 C372.

J.D. Horton, K. Kilakos, Minimum edge dominating sets, SIAM J. Disc Math. 6(3) (1993) 375-387.

One variant of edge dominating sets: Total edge dominating sets

• Let $G = (V, E)$ be a graph with vertex set V and edge set E. A subset $F \subseteq E$ is an edge total dominating set if every edge $e \in E$ is adjacent to at least one edge in F.

• The total edge domination number $\gamma_t'(G)$ of G is the minimum cardinality among all edge total dominating sets of G. [Kulli & Patwari, 1991]

• The TOTAL-EDGE-DOMINATING problem (ETDP) is to test whether $\gamma_t'(G) \leqslant k$ for an input graph G and an integer k.

V.R. Kulli, D.K. Patwari, On the edge domination number of a graph, in: Proceedings of the Symposium on Graph Theory and Combinatorics, Cochin, 1991, in: Publication, vol.21, Centre Math. Sci., Trivandrum, 1991, pp.75 C81.

Complexity of TEDP

• The TEDP is NP-complete for planar graphs with maximum degree three, and for undirected path graphs, a subclass of chordal graphs and a superclass of trees. [Zhang et al., 2014]
• A linear-time algorithm for solving TEDP in a tree. [Zhang et al., 2014]

Y. zhang, Z. Liao, L. Miao, On the algorithmic complexity of edge total domination, Theoretcial Computer Science, 557 (2014) 28-33.

Results on total edge domination

Algorithmic results

Theorem 2.1 (Pan et al, 2020)

The total edge dominating set problem for bipartite graphs with maximum degree 3 is NP-complete.

Z. Pan, Y, Yang, X. Li, S.-J. Xu, The complexity of total edge domination and some results on trees, J. Combin. Optim, 40 (2020) 571-589.

Bounds on $\gamma_t'(G)$ in terms of $\gamma'(G)$ on graphs G

- $\gamma'(G) \leqslant \gamma_t'(G) \leqslant 2\gamma'(G)$ for a general graph G, the bounds are sharp for trees.
- Characterizing the $(\gamma_t'(T) = 2\gamma'(T))$-trees.
- Characterizing the $(\gamma_t'(T) = \gamma'(T))$-trees.

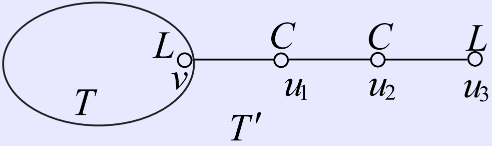
Characterizing the $(\gamma_t'(T) = 2\gamma'(T))$-trees

• We define the label of a tree T as a partition $L = (L_C, L_L)$ of $V(T)$. The label of a vertex v, denoted $l(v)$, is the letter L (or C) such that $v \in L_L$ (or $v \in L_C$). By a labelled P_4, we shall mean a P_4 with two non-leaf vertices labelled C and two leaves vertices labelled L.

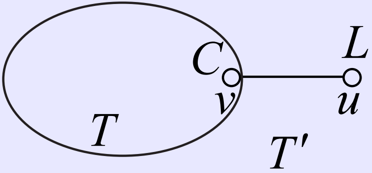
• Let $\mathcal{T}$ be the family of labelled trees T that contains a labelled P_4 and is under the two operations $\mathcal{O}_1$, $\mathcal{O}_2$ listed below: constructing a bigger tree from a smaller tree in $\mathcal{T}$.

Operation 1

- **Operation 1**: Let $T \in \mathcal{T}$ and v be a vertex of T with $l(v) = L$ such that: (1). each vertex labelled as C of distance 2 to v is adjacent to a leaf vertex; (2). for any $C - C$ edge wu of distance 1 to v, say v is adjacent to u, either u has a leaf neighbor other than v or all vertices in $N(w) - u$ are leaves. Construct a bigger tree T' in $\mathcal{T}$ from T and a labelled P_4 by identifying v and a leaf vertex of P_4, labelling the identified vertex as L and keeping the labels of the other vertices unchanged.



• **Operation 2:** Let $T \in \mathcal{T}$ and v a vertex of T with $l(v) = C$. Construct a bigger tree T' in $\mathcal{T}$ from T by adding a new vertex u adjacent to v, labelling u as L, keeping the labels of the other vertices unchanged.



Theorem for the $(\gamma_t'(T) = 2\gamma'(T))$-trees.

Theorem 2.2

A nontrivial tree T satisfies $\gamma_t'(T) = 2\gamma'(T)$ if and only if $T \in \mathcal{T}$ or T is a star.

Characterizing the $(\gamma_t'(T) = \gamma'(T))$-trees

• let T be a tree with $diam(T) = 4$, in which each edge is either a leaf edge or a support edge, we label support edges in T with S, leaf edges adjacent to at least two non-leaf-edges with L_2, other leaf edges with L_1.

• The edge labelling of a general tree can be recursively obtained by the following five operations.

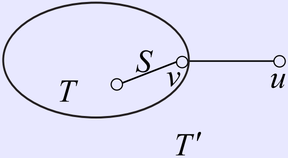
• Let $\mathcal{T}_t$ be the family of edge labelled trees T that contains a labelled tree with diameter 4 and is under the five operations $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$, $\mathcal{O}_4$, $\mathcal{O}_5$ listed below: constructing a bigger tree from a smaller tree in $\mathcal{T}_t$.

• According to the label of the associated edges of the vertex, we divide the vertex set into the following four subsets A_1, A_2, B, C listed below:

- $A_1 := \{v|$ Only one S-edge in $E(v)\}$;
- $A_2 := \{v|$ At least two S-edges in $E(v)\}$;
- $B := \{v|$ All edge in $E(v)$ are L_2-edges$\}$;
- $C = V - A_1 - A_2 - B$.

Operation 1

• **Operation 1**: Let $T \in \mathcal{T}_t$, v a vertex of T belonging to $A_1 \cup A_2$. Construct a bigger tree T' in $\mathcal{T}_t$ from T by adding a new vertex u adjacent to v. If $v \in A_1$, then labelling vu as L_1, adding u into C and keeping A_1, A_2, B unchanged. If $v \in A_2$, then labelling vu as L_2, adding u into B and keeping A_1, A_2, C unchanged.



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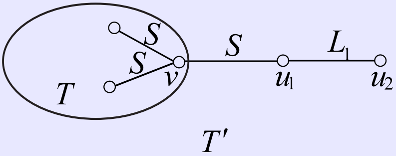
Results on total edge dominations

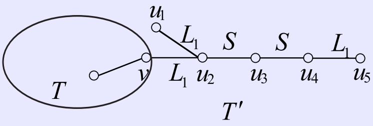
Results on semitotal edge dominations

- **Operation 2**: Let $T \in \mathcal{T}_t$, v be a vertex of T in A_2. Construct a bigger tree T' in $\mathcal{T}_t$ from T by adding two new adjacent vertices u_1, u_2, connecting v and u_1 and labelling vu_1 as S and u_1u_2 as L_1 (Obviously, $u_1 \in A_1$ and $u_2 \in C$).



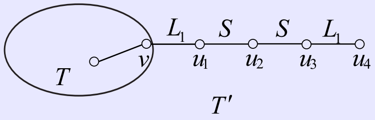
Operation 3

• **Operation 3**: Let $T \in \mathcal{T}_t$, $v \notin A_1$ be a vertex of T satisfying, in the case $v \in C$, that each L_1-edge in $E(v)$ is either adjacent to one leaf edge or contained by a $P_4 = vwxy$, whose edges are labelled as L_1, L_1, L_2 consecutively and all edges in $E(x)$ are L_2-edges except wx. Construct a bigger tree T' in $\mathcal{T}_t$ from T by adding a new path $u_1u_2u_3u_4u_5$ to join v and u_2, and labelling u_2u_3, u_3u_4 as S, and vu_2, u_1u_2, u_4u_5 as L_1, see Fig. **??**. (From the definition, $u_2, u_4 \in A_1$, $u_3 \in A_2$, $u_1, u_5 \in C$ and if $v \in B$, then v is moved from B to C.)

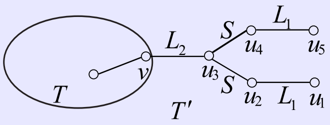


Operation 4

- **Operation 4**: Let $T \in \mathcal{T}_t$, $v \in B$ be a vertex of T. Construct a bigger tree T' in $\mathcal{T}_t$ from T by adding a new path $u_1u_2u_3u_4$ to join v and u_1, and labelling vu_1, u_3u_4 as L_1, and u_1u_2, u_2u_3 as S, see Fig. **??**. (Similarly, $u_1, u_3 \in A_1$, $u_2 \in A_2$, $u_4 \in C$, and v is moved from B to C.)



• **Operation 5**: Let $T \in \mathcal{T}_t$, v be a vertex of T. Construct a bigger tree T' in $\mathcal{T}_t$ from T by adding a new path $u_1u_2u_3u_4u_5$ to join v and u_3, and labelling vu_3 as L_2, u_1u_2, u_4u_5 as L_1, and u_2u_3, u_3u_4 as S, see Fig. **??**. (From the definition, $u_2, u_4 \in A_1$, $u_1, u_5 \in C$, $u_3 \in A_2$ and if $v \in B$, then v is moved from B to C.)



Theorem for the $(\gamma_t'(T) = \gamma'(T))$-trees.

Theorem 2.3

A nontrivial tree T satisfies $\gamma_t'(T) = \gamma'(T)$ if and only if $T \in \mathcal{T}_t$.

Results on semitotal edge dominations

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Background and Motivation

Results on total edge dominations

Results on semitotal edge dominations

Another variant of edge dominating sets: Semitotal edge dominating sets

- Semitotal edge dominating sets are squeezed between edge dominating sets and total edge dominating sets.
- A semitotal edge dominating set of a graph G is an EDS S such that for every edge $e \in S$, there exists an edge $e' \in S$ such that e either is adjacent to e' or shares a common neighbor edge with e'.
- The semitotal edge domination number $\gamma'_{st}(G)$ of G is the minimum cardinality among all semitotal edge dominating sets of G. [Zhu & Liu, 2019]
- $\gamma'(G) \leqslant \gamma'_{st}(G) \leqslant \gamma'_t(G)$ for general graph G.
- The SEMITOTAL-EDGE-DOMINATING problem (STEDP) is to test whether $\gamma'_{st}(G) \leqslant k$ for an input graph G and an integer k.

E.Q., Zhu, C.J. Liu, On the semitotal domination number of line graphs, Discrete Appl. Math. 254 (2019) 295-298.

NP-hard of the STEDP

Theorem 3.1 (Zhu & Liu, 2019)

The STEDP is NP-complete for planar graphs with maximum degree 4.

Theorem 3.2 (X. et al., 2020)

The problem of deciding whether $\gamma'(G) = \gamma'_{st}(G)$ is NP-hard in planar bipartite graphs with maximum degree 4.

Theorem 3.3 (X. et al., 2020)

The problem of deciding whether $\gamma'(G) = \gamma'_{t}(G)$ is NP-hard in planar graphs with maximum degree 4.

Theorem 3.4 (X. et al., 2021+)

The problem of deciding whether $\gamma'_t(G) = \gamma'_{st}(G)$ is NP-hard in planar bipartite graphs with maximum degree 4.

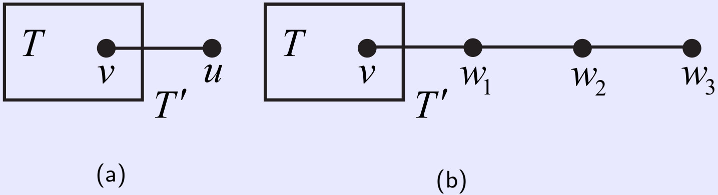
Charaterizations

- $\gamma'(G) \leqslant \gamma'_{st}(G) \leqslant \gamma'_t(G)$ for general graph G and the bounds are sharp.
- Characterizing $(\gamma' = \gamma'_t)$-trees in the previous section (equivalently, $(\gamma' = \gamma'_{st} = \gamma'_t)$-trees).
- Characterizing $(\gamma' = \gamma'_{st})$-trees?
- Characterizing $(\gamma'_t = \gamma'_{st})$-trees?

Charaterizing $(\gamma' = \gamma'_{st})$-trees

- Let $\mathcal{T}_s$ be the family of trees T containing a P_5, contructed inductively by the three operations $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$ listed below (i.e., constructing a bigger tree T' from a smaller tree T in $\mathcal{T}_s$).

Operation $\mathcal{O}_1$: Let v be a vertex of T satisfying $E(v) \cap S \neq \emptyset$ where S is some $\gamma'_{st}(T)$-set of T. Construct a bigger tree T' in $\mathcal{T}_s$ from one of two ways of following:
(a) Adding a vertex u and an edge vu, see Figure 1(a);
(b) Adding a path $P = w_1w_2w_3$ and joining v and w_1, see Figure 1(b).

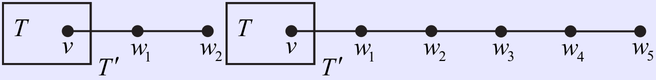


Operation $\mathcal{O}_2$

Operation $\mathcal{O}_2$: Let v be a vertex of T satisfying $\gamma'(T; E(v)) = \gamma'(T)$. Construct a bigger tree T' in $\mathcal{T}_s$ from one of two ways of following:

(a) Adding a path $P = w_1w_2$ and joining v and w_1, see Figure 1(c);

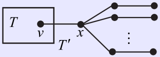
(b) Adding a path $P = w_1w_2w_3w_4w_5$ and joining v and w_1, see Figure 1(d).



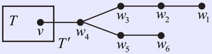
(c) (d)

Operation $\mathcal{O}_3$

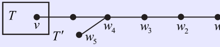
Operation $\mathcal{O}_3$: Let v be any vertex of T. Construct a bigger tree T' in $\mathcal{T}_s$ from one of three ways of following:
(a) Adding a subdivided star X with at least two leaves and joining the center vertex x of X and v, see Figure 1(e);
(b) Adding a path $P = w_1w_2w_3w_4w_5w_6$ and joining v and w_4, see Figure 1(f);
(c) Adding a path $P = w_1w_2w_3w_4w_5$ and subdividing the edge connecting v and w_4, see Figure 1(g).



(e)



(f)



(g)

Characterization of $(\gamma' = \gamma'_{st})$-trees

Theorem 3.5 (X. et al, 2020)

A tree is a $(\gamma' = \gamma'_{st})$-tree if and only if $T \in \mathcal{T}_s$.

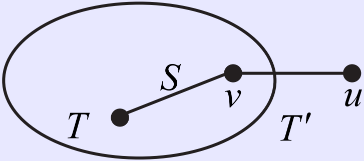
Z. Pan, X. Li, S.-J. Xu, Complexity and characterization aspects of edge-related domination for graphs, J Combin. Optim., 40 (2020) 757-773.

Characterizing $(\gamma'_t = \gamma'_{st})$-trees

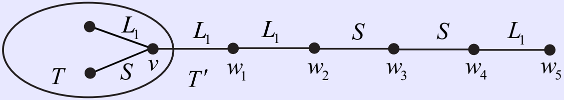
- Let $\mathcal{T}_{ts}$ be the family of edge-labelled trees containing edge-labelled trees with diameter 4 and constructed inductively by the six operations $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$, $\mathcal{O}_4$, $\mathcal{O}_5$, $\mathcal{O}_6$ listed below (i.e., constructing a bigger tree T' from a smaller tree T in $\mathcal{T}_s$).
- For the convenience of the following, an edge is labelled as S (resp. L_1, L_2) in $T \in \mathcal{T}_{ts}$ is called an S (resp. L_1, L_2)-edge, and denote by $D(T)$ the set of S-edges in *T*. First, according to the label of the edges incident with the vertex v in an edge-labelled tree $T \in \mathcal{T}_{ts}$, we partition the vertex set of T into the following four subsets A_1, A_2, B and C as follows:

$$A_1 := \{v | \text{ Only one } S\text{-edge in } E(v)\};$$
$$A_2 := \{v | \text{ At least two } S\text{-edges in } E(v)\};$$
$$B := \{v | \text{ All edge in } E(v) \text{ are } L_2\text{-edges}\};$$
$$C := V - A_1 - A_2 - B.$$

Operation $\mathcal{O}_1$: Let $T \in \mathcal{T}_{ts}$, v be a vertex of T in $A_1 \cup A_2$. Construct a bigger tree T' in $\mathcal{T}_t$ by adding a vertex u and an edge vu. If $v \in A_1$, then label vu as L_1; (By definition, u is in C, A_1, A_2, B are unchanged;) if $v \in A_2$, then label vu as L_2. Note that $u \in B$ and A_1, A_2, C are unchanged.

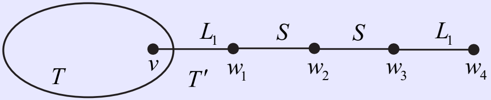


Operation $\mathcal{O}_2$: Let $T \in \mathcal{T}_{ts}$, v be a vertex of T in A_1 satisfying there is a $P_4 = vxyz$ started at v whose edges are labelled as S, S, L_1 consecutively and $E(z) - yz$ contains no S-edge and L_1-edge. Construct a bigger tree T' in $\mathcal{T}_s$ by adding a path $P = w_1w_2w_3w_4w_5$, joining v and w_1, labelling vw_1, w_1w_2, w_4w_5 as L_1, w_2w_3, w_3w_4 as S. By definition, $w_1, w_5 \in C$, $w_2, w_4 \in A_1$ $w_3 \in A_2$, and B is unchanged.



Operation $\mathcal{O}_3$

Operation $\mathcal{O}_3$: Let $T \in \mathcal{T}_{ts}$, v be a vertex of T in $B \cup C$. In the case $v \in C$, v satisfies the following conditions: (1), each L_1-edge in $E(v)$ is either adjacent to one leaf edge or contained by a $P_3 = vwx$, whose edges are labelled as L_1, L_1 and $E(x) - wx$ contains no S-edge and L_1-edge when the degree of the vertex in $N^2(v) \cap A_2$ is at least 3; (2), there is a $P_5 = vxyzw$ started at v whose edges are labelled as L_1, S, S, L_1 consecutively and $E(w) - wz$ contains no S-edge and L_1-edge when the degree of the vertex y in $N^2(v) \cap A_2$ is 2. Construct a bigger tree T' in $\mathcal{T}_s$ by adding a path $P = w_1w_2w_3w_4$, joining v and w_1 and labelling vw_1, w_3w_4 as L_1, and w_1w_2, w_2w_3 as S. From the definition, $w_1, w_3 \in A_1$, $w_2 \in A_2$, $w_4 \in C$, and v is moved from B to C if $v \in B$.

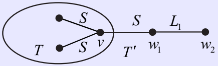


Operation $\mathcal{O}_4$

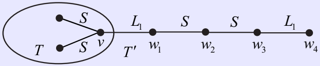
Operation $\mathcal{O}_4$: Let $T \in \mathcal{T}_{ts}$, v be a vertex of T in A_2 such that for each vertex u in $N(v) \cap A_1$, there is a $P_3 = vux$ started at v whose edges are labelled as S, L_1 consecutively and $E(x) - ux$ contains no S-edge and L_1-edge. Construct a bigger tree T' in $\mathcal{T}_t$ from one of two ways of following:

(1). Adding a new edge w_1w_2 to connect vertex v and w_1, labelling vw_1 as S and w_1w_2 as L_1. Obviously, $w_1 \in A_1$ and $w_2 \in C$ where A_2 and B are unchanged.

(2). Adding a $P_4 = w_1w_2w_3w_4$, connecting v and w_1, labelling vw_1, w_3w_4 as L_1, w_1w_2, w_2w_3 as S. By definition, $w_4 \in C$, $w_1, w_3 \in A_1$ and $w_3 \in A_2$.



(h)



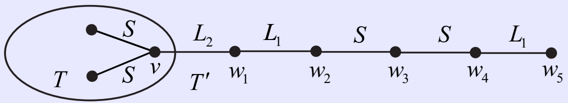
(i)

Operation $\mathcal{O}_5$

S.-J. Xu

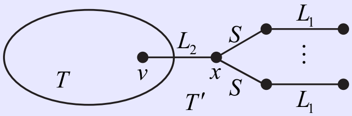
Background and Motivation
Results on total edge dominations
Results on semitotal edge dominations

Operation $\mathcal{O}_5$: Let $T \in \mathcal{T}_{ts}$, v be a vertex of T in A_2. Construct a bigger tree T' in $\mathcal{T}_t$ by adding a $P_5 = w_1w_2w_3w_4w_5$ and connecting v and w_1. If $v \in A_1$, then label vw_1 as L_1; if $v \in A_2$, then label vw_1 as L_2 and label w_1w_2, w_4w_5 as L_1, w_2w_3, w_3w_4 as S. By definition, $w_1, w_5 \in C$, $w_2, w_4 \in A_1$ and $w_3 \in A_2$.



Operation $\mathcal{O}_6$

Operation $\mathcal{O}_6$: Let $T \in \mathcal{T}_{ts}$, v be a vertex of T. Construct a bigger tree T' in $\mathcal{T}_s$ from T by adding a new subdivided star X with center vertex x and at lest two leaf vertices, see Fig. **??**. Let $N(x) = \{w_1, w_2, \ldots, w_l\}$ in X, and each $N(w_i) = \{x, z_i\}$ where $2 \leqslant l = i$. Then label edges vx as L_2, edges xw_i as S and $w_i z_i$ as L_1. From the definition, $w_i \in A_1$, $z_i \in C$, $x \in A_2$, if $v \in B$, then v is moved from B to C.



Characterization of $(\gamma'_t = \gamma'_{st})$-trees

Theorem 3.6 (Pan, X., 2021+)

A tree is a $(\gamma'_t = \gamma'_{st})$-tree if and only if $T \in \mathcal{T}_{ts}$.

Z. Pan, S.-J. Xu, Note on graphs with the total domination number equal to the semitotal domination number, Completed.

Thanks for your attention!